Chapter 11-2: Algebraic Expressions and Factorizations of Polynomials

Chapter Guidelines:

- Polynomial is a type of Algebraic Expression.
- Polynomials can be factored into prime polynomials.

1: Algebraic Expressions

Algebraic expression is a combination of integers, operators, constants, and variables. For example, $x + 3$ and $\sqrt[5]{2} - x^2$ are both algebraic expressions.

In the algebraic expression $(x + 2)$, x is the **variable**, 2 is a **constant**, and $+$ is an **operator**.

- A variable is an undefined, or unknown value, that is represented by some notations or alphabets. Keep in mind that not all alphabets represent a variable; for example, many physical constants like c and G are all constants represented by alphabets.
- A constant is a number which has a value that does not change; for example, number 1 will always have the numerical value 1, therefore the number 1 is a constant.
- An operator is a symbol that notates what arithmetic is demanded in an operation or algebraic expression. For example, the plus symbol "+", subtraction symbol "-", multiplication symbol "×", and division symbol "÷", are all operators.

For example: in the expression $\sqrt[5]{2} - x^2$, $\sqrt[5]{2}$ is a constant, x^2 is a variable, and "$-$" is an operator.

A specific type of algebraic expression that has <u>only integer exponents to its variable(s) and does not have variables in a fraction's denominator of the expression</u> is known as a "**polynomial**". In polynomials, each individual combination of variables, integers, and constants presented is known as a "**term**" in the polynomial and depending on the number of terms a polynomial has it is called a different name.
A polynomial with only one term it is called a **monomial**, for a polynomial with two terms it is called a **binomial**, and for a polynomial with three terms it is called a trinomial… the list goes on. A polynomial has a "**degree**", which is the numerical value of the largest exponent presented in the polynomial.
For example, the degree of polynomial $x^2 + 3x + 1 = 0$ is 2 because the largest integer exponent existent in this polynomial is "2".

Given the concepts of "degrees" and "terms", we can start referring the terms of polynomials by its degree, or the integer exponent it has. For example, in the polynomial $x^2 + 3x + 1 = 0$, x^2 is a second-degree term because its exponent is 2; $3x$ is a first-degree term because its exponent is

3; and 1 is the constant term because the term has no variables and consists solely of constant. You can also consider the constant term of a polynomial to be a "zeroth-degree term", since a constant term is also a product of its coefficient and zeroth powers of any variables in the polynomial.

Interestingly, both the sum and difference of polynomials would give a polynomial, and the product of polynomials would still be a polynomial. In the below examples, we will look at the algebra of polynomials (Excluding division, which we shall introduce in Unit 13, as divisions of polynomials may not produce a polynomial as a quotient).

$$\begin{array}{rr} & x^3+2x^2+5x+\ \ 9 \\ +) & x^2+4x+10 \\ \hline & x^3+3x^2+9x+19 \end{array}$$

We can add polynomials by performing addition on same-degree terms in the polynomials.
For example, the arithmetic for polynomial addition $(x^3 + 2x^2 + 5x + 9) + (x^2 + 4x + 10)$ is shown at left: the mathematician will add up terms with same degree in the polynomials to achieve a result.

Following the above:

$$(x^3 + 2x^2 + 5x + 9) + (x^2 + 4x + 10)$$
$$= x^3 + (2x^2 + x^2) + (5x + 4x) + (9 + 10)$$

In the previous step, we grouped all terms of same degree, and now we can add terms with same degree together.

$$x^3 + (2x^2 + x^2) + (5x + 4x) + (9 + 10)$$
$$= x^2 + 3x^2 + 9x + 19$$

We can operate subtractions with the same method as above:

$$\begin{array}{rr} & x^3+2x^2+5x+\ \ 9 \\ -) & x^2+4x+10 \\ \hline & x^3+\ x^2+\ \ x-\ \ 1 \end{array}$$

We subtract one polynomial from another by subtracting the terms of the subtrahend polynomial from the minuend polynomial, as shown left.
In horizontal expression the subtraction can be written as $(x^3 + 2x^2 + 5x + 9) - (x^2 + 4x + 10)$, which based on the rules of arithmetic is essentially an operation $(x^3 + 2x^2 + 5x + 9) - x^2 - 4x - 10$.

Following the above:

$$(x^3 + 2x^2 + 5x + 9) - (x^2 + 4x + 10)$$
$$= x^3 + (2x^2 - x^2) + (5x - 4x) + (9 - 10)$$

In the previous step, we grouped all terms of same degree, and now we can subtract terms with same degree together.

$$x^3 + (2x^2 - x^2) + (5x - 4x) + (9 - 10)$$
$$= x^2 + x^2 + x - 1$$

One can multiply polynomials using the distributive property, which states the following:

$$(a + b) \times c = ac + bc$$

By the same rule, we can multiply polynomials, for example: $(x+1)$ and $(x-1)$.

Based on distributive property:

$$(x+1)(x-1) = x(x-1) + 1(x-1)$$
$$x(x-1) + 1(x-1) = x^2 - x + x - 1 = x^2 - 1$$

Therefore, $(x+1)(x-1) = x^2 - 1$.

Summarize the above with a property:

$$(a+b)(c+d) = a(c+d) + b(c+d) = ac + ad + bc + bd$$

The process of multiplying polynomials and expressing its product as a polynomial is known as "**foiling**".
For example, on a lot of Math Team T-Shirts you will see the multiplication of two polynomials: $(P+L)(A+N) = PA + PN + LA + LN$.
The joke is that because you have foiled the multiplication of two binomials: $(P+L)\&(A+N)$, you have foiled your "PLAN" (which is the word variables of two polynomials spell).

We can also foil products of trinomials, and we will try foiling the product of two trinomials here:

$$(a+b+c)(d+e+f) = a(d+e+f) + b(d+e+f) + c(d+e+f)$$
$$= ad + ae + af + bd + be + bf + cd + ce + cf.$$

2: Factorization of Polynomials

Factorization is the action of expressing an algebraic expression, or a numeric expression at most times, as a product of other expressions. Algebraic expressions can be factorized into product of numerical and algebraic expressions, while numeric expressions can only be factorized into a product of numerical expressions.

That being said, we can also factorize a polynomial, which is an algebraic expression. The five most common methods of doing so are: factoring by common factors, factoring by grouping, factoring by cross-multiplications, factoring by substitution, and factoring by algebraic shortcuts. Most factorizations require mathematicians to employ multiple methods, sometimes repetitively, to completely factorize a polynomial.

When factoring a polynomial, we wish to factor it into products of **prime polynomials**, which are polynomials that cannot be factored into lower-degree polynomials.
It is worth noticing that not all binomials are prime polynomials, as it is a common misunderstanding. For example, the binomial $(x^2 - 1)$ in the above example can be factored as a product of $(x+1)$ and $(x-1)$.

1: Factoring by Common Factors

Now we can provide a description of this method "**Factoring by Common Factors**":

Factoring a polynomial by its common factor is a method of factorization that first finds the greatest common factor (n) of coefficients, then express the polynomial as a product of n the greatest common factor and expressions.

Let us look at this polynomial: $3x + 6$, and think about how we can factor this polynomial.

The first step of factoring is to make a brief analysis about this polynomial.
For example, if we look at the coefficients of each term, we can see the coefficient of the first-degree term to be 3, and the constant term's to be 6.
Since the greatest common factor of these two coefficients are 3, we can express this polynomial as the product of numerical expression (3) and binomial (algebraic expression) $(x + 2)$.
We can verify our decision by multiplying them back together: $3(x + 2) = 3x + 6$.

We also mentioned that we factor a polynomial until it has been factored into a product of prime polynomials. This first-degree polynomial $(x + 2)$ cannot be factored into lower-degree polynomials. In this case, $3(x + 2)$ is the prime factorization of the polynomial $3x + 6$.

And we can look further to an extended example.
Let us factor the polynomial $3x^2 + 6x$.

(First, we can observe the coefficients of this binomial, which are 3 and 6. The greatest common factor of both numbers is 3, so we may as well factor this polynomial into $3(x^2 + 2x)$. Then, we may look at the binomial $(x^2 + 2x)$ and should observe that they have a common factor of "x", a variable, which would lead to the factorization $(x^2 + 2x) = x(x + 2)$.
As we know $(x + 2)$ is a prime polynomial, we can conclude that the prime factorization of the polynomial $(3x^2 + 6x)$ is then $3x(x + 2)$.

2: Factoring by Grouping

Now we provide a summarizing description of this factorization method:

Factoring by Grouping is a method of factorization that classifies different terms of polynomials into groups with common factors and then use the distributive property to result the prime factorization of polynomials.

It is time to look at a third-degree polynomial.
To factor the polynomial $(x^3 + x^2 + x + 1)$, we should start observing this polynomial.

First of all, this polynomial's coefficients are all 1, and we can group this polynomial in 2 ways:

$Solution\ A$: $x^3 + x^2 + x + 1 = (x^3 + x) + (x^2 + 1)$

$Solution\ B$: $x^3 + x^2 + x + 1 = (x^3 + x) + (x^2 + 1) = (x^3 + x^2) + (x + 1)$

By factoring with common factors, we can further factor this polynomial into the following forms:

$Solution\ A$: $(x^3 + x) + (x^2 + 1) = x(x^2 + 1) + 1(x^2 + 1) = (x + 1)(x^2 + 1)$

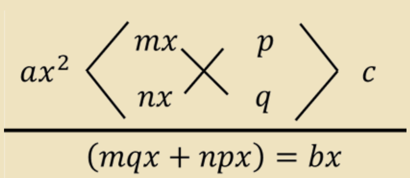
$Solution\ B$: $(x^3 + x^2) + (x + 1) = x^2(x + 1) + 1(x + 1) = (x^2 + 1)(x + 1)$

Either factorization would lead to the same result.

As both $(x + 1)$ and $(x^2 + 1)$ are prime polynomials, we can conclude that the prime factorization for polynomials $x^3 + x^2 + x + 1$ is $(x^2 + 1)(x + 1)$.

3: Factoring by Cross-Multiplication

First and foremost, this **cross-multiplication** does not relate to the cross-multiplication of denominators and numerators in fractions but relates to the multiplication of two binomials. Factoring by cross-multiplication is a factorization method that factors a trinomial into two binomials.

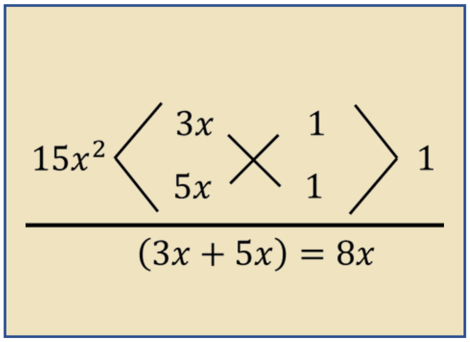


Say we have a polynomial $ax^2 + bx + c$ that can be factored into $(mx + p)(nx + q)$, this polynomial can also be foiled into $mnx^2 + (mq + np)x + pq$.
Cross multiplication relies on the above fact that the first-degree terms of such polynomial above is created by multiplying terms across different prime polynomials.
This can be shown in left, where terms across straight ways of the cross multiply, and the sum of two products form the first degree term of this second-degree polynomial.

Given what cross multiplication is, let us discuss how did cross multiplication help factorizing polynomials.

The first step in with cross multiplication factoring is to factorize the second degree term and the constant term, because we know that some factor of second-degree term and constant term multiplied together will give parts of first-degree term. That being said, we are finding two pairs of factors, each consisting of one second-degree term's factor and one constant term's factor, that will sum up to the first-degree term of polynomial.
The second step is to multiply the factors of second-degree and constant terms in a pair and calculate the sum of two products from two pairs we selected earlier. If the sum is not equivalent to the first-degree term of the polynomial, experiment another pair; if otherwise, go on to step 3.
It is worth noting that cross multiplication with only integer factors from constant term sometimes do not work, either because the polynomial itself is already prime or because the **roots** (value of variable) of the polynomial are irrational numbers.
The third step is to produce prime polynomials with factors in pre-elected pairs by adding them together.

Now, we may look at the example $(15x^2 + 8x + 1)$.

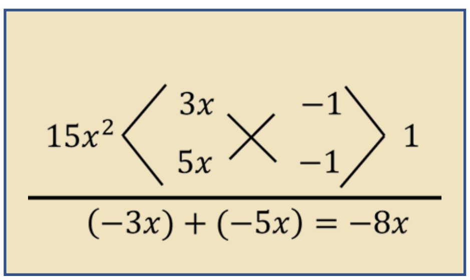


The process of cross multiplication for polynomials is presented at left.
We first factorize the highest-degree term and lowest-degree of the trinomial into two monomials and put them at each endpoint of the cross. If with the right factorization, the sum of products of monomials at the opposite endpoint of each line segment in the cross will be the second term of the polynomial. Two individual sums of two horizontally across monomials are the two factors of the trinomial.

Mathematicians will test multiple combinations of factorizations of largest-degree and smallest-degree term until the sum of products of corresponding monomials match the second term of the factored polynomial.
At above, we see that since$3x \times 1 + 5x \times 1 = 8x$, the first-degree term of trinomial, we know that this set of factorizations is the right factorization.
The factorization is then completed: $15x^2 + 8x + 1 = (3x + 1)(5x + 1) = 15(x + \frac{1}{3})(x + \frac{1}{5})$.
We made the final result of factorization that way because when factorizing polynomials, we want to practice a good habit of making the coefficients of our variable zero, as it would make later assignments easier when we need to inspect more properties of polynomials.

So, in summary, **Factoring by Cross-Multiplication** is a method to factor a trinomial into two binomials using cross-multiplication, the process introduced above based on multiplicative rules of binomials.

We may then look at our second example: $15x^2 - 8x + 1$



This time, there are two possible sets of correct factorizations: $(-5x + 1)(-3x + 1)$ & $(5x - 1)(3x - 1)$. To simplify the final result of factorization, we usually want coefficients of variable terms to be 1, so we will just go with $15(x - \frac{1}{3})(x - \frac{1}{5})$ since both factorizations are algebraically equivalent once they are foiled.

4: Factoring by Substitution

Substitution is a very practical technique in algebra.
Substitution is a method of defining an algebraic expression as a variable to simplify an equation and solve it based on the substituting variable to simplify the process of solving or factoring an expression.

Factoring by Substitution is a method of factorization by substituting a complex algebraic expression with a variable and reducing it into the original algebraic expression it resembles after prime factorization has been accomplished.

Using Substitution while factorization can sometimes provide benefits, such as well efficiency. For example, it would be very a very irritating process when we are factoring the polynomial: $(5x+1)^2 - 3(5x+1) + 2$, since we would have to foil the whole polynomial to factor it in the three methods we learned above.

However, if we use substitution on the polynomial instead and set up a substitution equation, for example: $y = 5x + 1$, then the above polynomial would be simplified into $y^2 - 3y + 2$, which we can use factorization by cross-multiplication to factor it. Its prime factorization would be $(y-1)(y-2)$.
Well, substitution just made your life easier, albeit a tad.

Then, we can convert y in the expression into $5x + 1$ since we have previously set up the premise that $y = 5x + 1$ when we started the substitution.
We will then attain the prime factorization:

$$(5x+1)^2 - 3(5x+1) + 2 = 5x(5x-1) = 25(x)(x - \frac{1}{5})$$

To summarize the entire operation above:

> Example: Factorize the polynomial: $(5x+1)^2 - 3(5x+1) + 2$.
> Step 1. Substitution: $y = 5x + 1 \rightarrow (5x+1)^2 - 3(5x+1) + 2 = y^2 - 3y + 2$.
> Step 2. Cross Multiplication: $y^2 - 3y + 2 = (y-2)(y-1)$
> Step 3. Replace y in the expression with $5x + 1$, so $(y-2)(y-1) = 5x(5x-1)$.
> Step 4. Simplify the expression: $5x(5x-1) = 25(x)(x - \frac{1}{5})$

Again, we can also foil out the whole equation into $25x^2 - 5x$, then factor it by using "Factoring with Common Factors" and get the same result as the former method does.
To summarize the entire operation above:

> Example: Factorize the polynomial: $(5x+1)^2 - 3(5x+1) + 2$.
> Step 1. Foil the polynomial.
> $$25x^2 + 10x + 1 - 15x - 3 + 2 = 25x^2 - 5x$$
> Step 2. Factorize by Common Factor. $25x^2 - 5x = 5x(5x-1)$.
> Step 3. Simplify the expression: $5x(5x-1) = 25(x)(x - \frac{1}{5})$.

This example also shows us an issue in factorization of polynomials, which is that different methods have different degrees of efficiency in different factorization problems. It requires some practice to be able to identify the most efficient factoring method for a question.
That being said, not every polynomial can be factorized with the same factorization method.

5: Factoring by Algebraic Shortcuts

Algebraic Shortcuts are algebraic expressions that can be foiled into specific polynomials we can use in factorizations. It is called a "shortcut" because if one can memorize the foiled form of such algebraic expressions, it dramatically (sometimes) reduces the workload and time of factoring a polynomial, therefore increasing the efficiency.

That being said, below is a list of useful algebraic shortcuts:

Polynomial	Shortcut (Factorization)	Polynomial ($y = 1$)	Shortcut (Factorization)
$x^2 + 2xy + y^2$	$(x + y)^2$	$x^2 + 2x + 1$	$(x + 1)^2$
$x^2 - 2xy + y^2$	$(x - y)^2$	$x^2 - 2x + 1$	$(x - 1)^2$
$x^2 - y^2$	$(x + y)(x - y)$	$x^2 - 1$	$(x + 1)(x - 1)$
$x^3 + y^3$	$(x + y)(x^2 - xy + y^2)$	$x^3 + 1$	$(x + 1)(x^2 - x + 1)$
$x^3 - y^3$	$(x - y)(x^2 + xy + y^2)$	$x^3 - 1$	$(x - 1)(x^2 + x + 1)$
$x^3 + 3x^2y + 3xy^2 + y^3$	$(x + y)^3$	$x^3 + 3x^2 + 3x + 1$	$(x + 1)^3$
$x^3 - 3x^2y + 3xy^2 - y^3$	$(x - y)^3$	$x^3 - 3x^2 + 3x - 1$	$(x - 1)^3$

Do not just urge yourself to remember all of the shortcuts at once. Remember them as you practice factorizations and use them. In other words, remembering all of the shortcut takes more than recitation. You need to apply them to problems to really remember and utilize the algebraic shortcuts efficiently.

Noe, let us factor $(x^3 - x^2 - x + 1)$ as an example:

Step 1: Using factoring by grouping, we find the polynomial to also be $(x^3 - x) - (x^2 - 1)$. Then, by using factoring by common factors, we simplify this expression into $(x - 1)(x^2 - 1)$.

Step 2: Apparently, the prime factorization has been achieved since it looks like we have only prime polynomials in our factorization, but with the list of algebraic shortcuts we see above, we cannot help but to notice that $(x^2 - 1)$ is in fact a product of $(x - 1)$ and $(x + 1)$.
Using factoring by algebraic shortcuts, the prime factorization of this third-degree trinomial should be $(x^3 - x^2 - x + 1)$ should actually be $(x - 1)^2(x + 1)$.
We can use exponents to simplify our prime factorizations of polynomials; therefore:
$(x^3 - x^2 - x + 1) = (x - 1)^2(x + 1)$.

Factoring by Algebraic Shortcuts is the method of factorization that factorizes polynomials using algebraic shortcuts to factorize a specific polynomial into smaller factor polynomials.

We can look at another example: $(2x^2 + 4x + 2)$.

We can first use the method factoring by common factors to factorize this polynomial into $2(x^2 + 2x + 1)$, and since $(x^2 + 2x + 1)$ is equivalent to $(x + 1)^2$ (as shown in the algebraic shortcut), this polynomial can also be prime factorized into $2(x + 1)^2$.
Therefore, $(2x^2 + 4x + 2) = 2(\mathrm{x} + 1)^2$.

6: When Should I Stop Factoring?

You can stop factoring after all factors you have are all prime polynomials with constant terms being integers or zeros, regardless of the coefficient of non-constant-degree terms.

Practice Questions (No CALC)

Part I. Provide an answer to the given arithmetic of polynomials.

*Ex. 8, 12-19 will be revisited in C11-3 and C12-1 for an easier solution.

*Ex. 23, 25 will be revisited in Chapter: Pascal Triangle for an easier solution.

Operation	Answer
(1) $(\boldsymbol{x+6})\times(\boldsymbol{x+2})$	$x^2+8x+!2$
(2) $(\boldsymbol{x+6})\times(\boldsymbol{x-2})$	$x^2+4x-12$
(3) $(\boldsymbol{-x+6})\times(\boldsymbol{-x-2})$	$x^2-8x-12$
(4) $(\boldsymbol{3x+8})\times(\boldsymbol{5x-7})$	$15x^2+19x-56$
(5) $(\boldsymbol{2x-1})\times(\boldsymbol{2x-1})\times(\boldsymbol{2x-1})$	$8x^3-12x^2+6x-1$
(6) $(\boldsymbol{4x-9})\times(\boldsymbol{4x-9})$	$16x^2-72x+81$
(7) $(\boldsymbol{4x-9})\times(\boldsymbol{4x+9})$	$16x^2-81$
(8) $(\boldsymbol{x-3})^{\boldsymbol{4}}\times(\boldsymbol{4x^2-16})$	$4x^6-48x^5+200x^4-240x^3-540x^2+1728x-1296$
(9) $(\boldsymbol{3x^2+1})\times(\boldsymbol{2x-1})$	$6x^3-3x^2+2x-1$
(10) $(\boldsymbol{x^2+2x})\times(\boldsymbol{x-\sqrt{8}})$	$x^3-2\sqrt{2}x^2+2x^2-4\sqrt{2}x$
(11) $(\boldsymbol{x^2+3x+2})\times(\boldsymbol{x+1})$	x^3+4x^2+5x+2
(12) $(\boldsymbol{4x^2-9x+13})\times(\boldsymbol{\sqrt{13}x-9})$	$4\sqrt{13}x^3-\left(9\sqrt{13}+36\right)x^2+\left(13\sqrt{13}+81\right)x-117$
(13) $(\boldsymbol{x^2-9x+27})\times(\boldsymbol{-x+2})$	$-x^3+11x^2-45x+54$
(14) $(\boldsymbol{-x+3})\times(\boldsymbol{x^2+5})$	$-x^3+3x^2-5x+15$
(15) $(\boldsymbol{x^2+5x+12})\times(\boldsymbol{x^3-7})$	$x^5+5x^4+12x^3-7x^2-35x-84$
(16) $(\boldsymbol{x+5})\times(\boldsymbol{x+8})\times(\boldsymbol{x-9})$	$x^3+4x^2-77x-360$
(17) $(\boldsymbol{2x^2-9})\times(\boldsymbol{3x+\sqrt{12}})$	$6x^3+4\sqrt{3}x^2-27x-18\sqrt{3}$
(18) $(\boldsymbol{-3x^2+5x-3})\times(\boldsymbol{x+5})\times(\boldsymbol{2x-1})$	$-6x^4-17x^3+54x^2-52x+15$
(19) $(\boldsymbol{3x^2+\sqrt{8}x-1})\times(\boldsymbol{2x^2-5x-9})\times(\boldsymbol{9x^2-\sqrt{5}x+2})$	Coefficients for terms: $\begin{cases} 0th=18 \\ 1st=-9\sqrt{5}-36\sqrt{2}+10 \\ 2nd=18\sqrt{10}-5\sqrt{5}-20\sqrt{2}+23 \\ 3rd=10\sqrt{10}+29\sqrt{5}-154\sqrt{2}+15 \\ 4th=-4\sqrt{10}+15\sqrt{5}-90\sqrt{2}-249 \\ 5th=-6\sqrt{5}+36\sqrt{2}-135 \\ 6th=54 \end{cases}$
(20) $\left(\boldsymbol{x+\sqrt{2}}\right)^{\boldsymbol{2}}\times\left(\boldsymbol{x-\sqrt{2}}\right)^{\boldsymbol{2}}$	x^4-4x^2+4
(21) $\left(\boldsymbol{x+\sqrt{2}}\right)^{\boldsymbol{3}}\times\left(\boldsymbol{x-\sqrt{2}}\right)^{\boldsymbol{3}}$	$x^6-6x^4+12x^2-8$
(22) $\left(\boldsymbol{x+\sqrt{2}}\right)^{\boldsymbol{3}}\times\left(\boldsymbol{x-\sqrt{2}}\right)^{\boldsymbol{2}}$	$x^5+2\sqrt{2}x^4-4x^3-4\sqrt{2}x^2+4x+4\sqrt{2}$
(23) $\left(\boldsymbol{x-\sqrt{2}}\right)^{\boldsymbol{4}}$	$x^4-4\sqrt{2}x^3+12x^2-8\sqrt{2}x+4$
(24) $(\boldsymbol{-2x^2+\sqrt{10}})\times(\boldsymbol{5x^2+13})$	$-10x^4+\left(5\sqrt{10}-26\right)x^2+13\sqrt{10}$

(25) $(\boldsymbol{x}+\mathbf{1})^{\mathbf{16}}(\boldsymbol{x}-\mathbf{1})^{\mathbf{16}}$	$x^{32}-16x^{30}+120x^{28}-560x^{26}+1820x^{24}-4368x^{22}+8008x^{20}-11440x^{18}+12870x^{16}-11440x^{14}+8008x^{12}-4368x^{10}+1820x^{8}-560x^{6}+120x^{4}-16x^{2}+1$ (Notice the symmetric pattern?)

Part II. Provide an answer and a factored answer to the given arithmetic of polynomials.

Operation	Answer	Factored Answer
(1) $(\boldsymbol{x^2+2x})+(\boldsymbol{x^3+8})$	x^3+x^2+2x+8	$(x+2)(x^2-x+4)$
(2) $(\boldsymbol{x^2-2x})-(\boldsymbol{x^3-8})$	$-x^3+x^2-2x+8$	$-(x-2)(x^2+x+4)$
(3) $(\boldsymbol{x^3-2})+(\boldsymbol{x^4-2x})$	x^4+x^3-2x-2	$(x+1)(x^3-2)$
(4) $\boldsymbol{2x(x-1)-(x-1)^2}$	x^2-1	$(x-1)(x+1)$
(5) $(\boldsymbol{4x^2+59})-[\boldsymbol{3x^2-19(x+1)}]$	$x^2+19x+78$	$(x+13)(x+6)$
(6) $(\boldsymbol{4x^2-5x-10})-(\boldsymbol{5x^2-x-6})$	$-x^2-4x-4$	$-(x+2)^2$
(7) $(\boldsymbol{11x^2+5})-(\boldsymbol{3x^4-8x^2+11})$	$-3x^4+19x^2-6$	$-3(x^2-\frac{1}{3})(x^2-6)$
(8) $(\boldsymbol{11x^3-5x})-(\boldsymbol{3x^2+10x+x^2})$	$11x^3-4x^2-15x$	$11(x-\frac{15}{11})(x+1)$
(9) $\boldsymbol{4(2x-1)^2-(4x-2)^2+(8x-4)^4}$	$4096x^4-8192x^3+6144x^2-2048x+256$	$4096\left(x-\frac{1}{2}\right)^4$
(10) $(\boldsymbol{4-8x})+(\boldsymbol{2x+3})^{\mathbf{2}}-(\boldsymbol{3x^3+6})$	$-3x^3+4x^2+4x+7$	$-3(x+\frac{7}{3})(x^2+x+1)$
(11) $\boldsymbol{2(x+2)-(5x^2+2)+(4x-3)}$	$-5x^2+6x+1$	$-5(x-\frac{1}{5})(x-1)$
(12) $(\boldsymbol{3x^3-x+27})-(\boldsymbol{2x^3-x^2+26})$	x^3+x^2-x-1	$(x+1)(x-1)^2$
(13) $\left(\boldsymbol{x+\sqrt{2}}\right)^{\mathbf{2}}+\left(\boldsymbol{x-\sqrt{2}}\right)^{\mathbf{2}}$	$2x^2+4$	$2(x^2+2)$
(14) $\left(\boldsymbol{x+\sqrt{2}}\right)^{\mathbf{2}}-\left(\boldsymbol{x-\sqrt{2}}\right)^{\mathbf{2}}$	$4\sqrt{2}x$	$4\sqrt{2}x$
(15) $\left(\boldsymbol{x+\sqrt{2}}\right)^{\mathbf{3}}+\left(\boldsymbol{x-\sqrt{2}}\right)^{\mathbf{3}}$	$2x^3+12x$	$2x(x^2+6)$
(16) $\left(\boldsymbol{x+\sqrt{2}}\right)^{\mathbf{3}}-\left(\boldsymbol{x-\sqrt{2}}\right)^{\mathbf{3}}$	$6\sqrt{2}x^2+4\sqrt{2}$	$2\sqrt{2}(3x^2+2)$
(17) $\left(\boldsymbol{x+\sqrt{2}}\right)^{\mathbf{4}}+\left(\boldsymbol{x-\sqrt{2}}\right)^{\mathbf{4}}$	$2x^4+24x^2+8$	$2(x^4+12x^2+4)$
(18) $\left(\boldsymbol{x+\sqrt{2}}\right)^{\mathbf{4}}-\left(\boldsymbol{x-\sqrt{2}}\right)^{\mathbf{4}}$	$8\sqrt{2}x^3+16\sqrt{2}x$	$8\sqrt{2}x(x^2+2)$

Part III. Provide a factored answer to the given arithmetic of polynomials.

Operation	Factored Answer
(1) $\boldsymbol{0.4x^2-2.2x+1}$	$0.4(x-5)(x-0.5)$
(2) $\boldsymbol{1.2x^2-2x+0.3}$	$1.2(x-1.5)(x-\frac{1}{6})$
(3) $\boldsymbol{-x^2+5xy-6y^2}$	$-(x-3y)(x-2y)$
(4) $\boldsymbol{12(x-2y)^2+21(x-2y)(4x+5y)+9(4x+5y)^2}$	$240(x+\frac{3}{5}y)(x+\frac{7}{16}y)$

(5) $\mathbf{5x^3y + 10x^2y^2 - 15xy^3}$	$5xy(x-y)(x+3y)$
(6) $\mathbf{4(x^3-8) - 2(x^2-4) + 6(x^4-16) + (2x-4)(x-9)(x+3)}$	$6(x-2)(x+1)^3$
(7) $\mathbf{3x^{10} + 5x^5y^5 + 2y^{10}}$	$(3x^5+2y^5)(x+y)(x^4-x^3y+x^2y^2-xy^3+y^4)$
(8) $\mathbf{(x^2+3x+1)(x^2+3x+10)+8}$	$(x+1)(x+2)(x^2+3x+9)$
(9) $\mathbf{(x+y)^2 - 10(x^2-y^2) + 9(x-y)^2}$	$-16y(x-\frac{5}{4}y)$