Chapter 10-1: Type of Numbers and Simplification of Expressions

Chapter Guidelines:

- The numbers we used in math classes belongs to the system of Complex Numbers.
- Expressions are combinations of numbers, variables, and non-comparing-purpose operators.

1: Types of Numbers

In the field of algebra, mathematicians classify numbers into different categories.
Below is a table of general categories of numbers:

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Layer	Number Type (Set Code)	Description
1	Complex Numbers ($\mathbb{C}$)	Complex numbers are numbers in format $a + bi$, where a and b are any number on the number line and i represents square root of -1, to be written as $\sqrt{-1}$. Examples include: $2 + i$, $\sqrt{2} - \sqrt{3}i$, $41 + 0i$. They are divided into 2 subcategories known as “real numbers” and “imaginary numbers”.
2	Real Numbers ($\mathbb{R}$)	Real numbers are numbers that can be found on a number line.
	Imaginary Numbers	Imaginary numbers are numbers that cannot be found on a number line, such as $\sqrt{-1}$.
3	Algebraic Real Numbers	Transcendental numbers are numbers that can be a solution to a specific type of equation. This specific type of equation must only have one variable and only have rational coefficients, which cannot all be 0. Algebraic Real Numbers are all rational.
	Transcendental Numbers	Transcendental numbers are numbers that cannot be a solution to a specific type of equation, such as the value of π. This specific type of equation must only have one variable and only have rational coefficients, which cannot all be 0. Transcendental numbers are all irrational.
4	Rational Numbers ($\mathbb{Q}$)	Numbers that can be expressed as a fraction of integers.
	Irrational Numbers	Numbers that can’t be expressed as a fraction of integers.

2: Types and Forms of Expressions

An **expression** is a combination of variables, numbers, and operators (excluding the symbols $=$ and $\neq$).

For example: $2x + 5y$ is an expression with variables x, y; numbers 2, 5; an operator $+$.

It is worth noting that $2x + 5y = 1$ is not an expression, but an equation. An expression would not have symbols that compare two expressions, such as: $=$, $\neq$, $>$… etc.

There are two types of expressions: algebraic expressions and numerical expressions.

Algebraic expression is a combination of integers, operators, constants, and variables. For example, $x + 3$ and $2x + 5y$ are both algebraic expressions.

Then, **numerical expressions** refer to expressions that does not include any types of variable. For example, $1 + 3$ and $2 \times 4 \div 5 + 2$ are both numerical expressions. Occasionally we see numerical expressions without any operators, and we are to discuss those forms of expressions below.

1: Exponential Form

Mathematicians developed a use expression to represent a certain value of number in case it is too long to record. For example, if we want to multiply number 2 by itself 3 times, then instead of writing out the entire expression, $2 \times 2 \times 2$, we can record it as 2^3.

This form of expression that represents a value (either number or variable) multiplying itself for a number of times is called **exponential expression**.

And we will analyze the exponential expression 2^3 in below area:

The exponential form 2^3 has numbers written in conventional method, which is the number 2 in the expression, and a number written as a superscript, which is the number 3 in the expression that is written above the conventionally written number and is smaller.

In the above method to write exponential expressions, the conventionally written number (number 2 in this case) is known as the **base**, which is the number that multiplies itself, while the superscripted value (number 3 in this case) would be called the **exponent**, the number of times the base multiplies itself by.

Exponential expressions can also be written without a superscripted exponent. This expression x^3 could also be written as x^3 (where the symbol ^ is pronounced as lambda). Because there are multiple ways to write and pronounce an exponential expression, it is better to remember the definition of base and exponents by their function than by their positions in an expression or pronunciation.

There is also a basic terminology that one must know when talking about expressions: terms. A "**term**" means an item in an expression. For an expression involving numerous exponential

expressions (or expressions with exponential form), all exponential expressions with the same variable as their base are known to be a type of term. For example, x^3 and x^2 are all "x-terms", while y^4 and y^7 are "y terms".
The method of combining terms of same types will be introduced after the next paragraphs.

The mathematical expression of multiplying number x by itself a times is noted as x^a, but it can also be written as "x^a". This expression x^a is pronounced "x to the power of a^{th}".
If $a = 2$, where the expression would be x^2, we can call the expression "x squared" instead of "x to the second power". By the same logic, if $a = 3$, where the expression would be x^3, then we can call the term "x cubed" instead of "x to the third power".

The advantage in using exponential expression is convenience.
When a mathematician is asked to write an expression presenting the value of x multiplying itself for 35 times, the mathematician can write out the exponential form of that expression: x^{35}, rather than writing out 35 xs and 34 multiplication operators $\boxed{\times}$.

These are some properties of exponents:

1. If $x \neq 0$ and x^n is defined, then $x^{-n} = \frac{1}{x^n}$
2. If a^n and b^n are defined, then $a^n \times b^n = (ab)^n$
3. If $b \neq 0$ and a^n and b^n are defined, then $a^n \div b^n = (a \div b)^n$
4. If $a \neq 0$ and $b \neq 0$, then $a^n + b^n \neq (a + b)^n$. I put in this one because this is a common misconception.
5. If a^n and a^k are defined, then $a^n \times a^k = a^{n+k}$.
6. If $a \neq 0$ and a^n and a^k are defined, then $a^n \div a^k = a^{n-k}$.

In fact, we can generate one more property of exponents, but we will introduce several mathematical notations here to save space talking about the property:

Introducing Symbol: ∀.
The symbol ∀ directly translates to "for all".
For example, it can be used in a statement: "$\forall\ x \in \mathbb{Q}$, x can be written as a fraction of integers."
The above statement is translated as: "For all x that are in the set of all rational numbers, x can be written as a fraction of integers."
For another example, this symbol can be used in a statement: "$\forall\ x \in \mathbb{N}$, $x^2 > x$."
This translates as: "For all x that are in the set of all natural numbers, x^2 is larger than x."

Introducing Symbol: Set-builder notation.

Set-builder notation is often used to define the range for a statement or an equation to be true.

For those who went through "Trigonometric Functions" in Volume One of this book, you can think of it as a notation to define the range of input for a function to be true.
The above definitions are general until we learn about functions and give it a better definition, which would be at Unit 12.

The set-builder notation follows the format: $\{symbol \mid range\ and\ condition\ of\ symbol\}$.
For example, the set $\{x|x \in \mathbb{R}, |x| \leq 2\}$ can be translated literally as "The set of all x such that x is in the set of all real numbers (simply put, x is a real number) and its absolute value is less or equal to 2.

To avoid confusion about the usage of symbol "|" in the set-builder notation, the "|" symbol that separates symbol and its condition would often be replaced by a colon ":", which means the above set-builder notation could also be written as $\{x: x \in \mathbb{R}, |x| \leq 2\}$.

Here is an example of using the above notations to express a property.

Property: $\forall\ x \in \{x|x \in \mathbb{R}, x \neq 0\}, x^0 = 1$.
It literally translates to: "For all x that belongs to the set of xs that 'is made of all numbers (or xs) that are real numbers and does not equal to 0', x to the zeroth power equals 1."

We will explain this above property in two parts: why this property works (a proof) and why x was stated to not equal 0.

Part I: The proof. I will present the proof in a generally informal/oral way to assist my explanation and readers' comprehension.

First, I will create a fraction $\frac{x^a}{x^a}$, where $a \in \mathbb{R}$.
This fraction can also be expressed as $x^a \div x^a$, and because this expression is equal to a nonzero number dividing by itself, it is equal to 1.
We know x^a is nonzero because x is stated to be a nonzero real number.
Then, according to the property of exponents where "$\forall\ a, b, n \in \mathbb{R}$, $a^n \div a^k = a^{n-k}$", we can then produce an expression $x^a \div x^a = x^{a-a} = x^0$.
$x^a \div x^a = 1$ and $x^a \div x^a = x^0$; thus, $\forall\ x \in \{x|x \in \mathbb{R}, x \neq 0\}, x^0 = 1$.

Part II: Why $x \neq 0$?

Mathematicians have long debated about the value of 0^0, and although many mathematicians agree that $0^0 = 1$, this identity still changes as we shift between different fields of mathematics.
Another popular perspective states that 0^0 is undefined, and some would even consider it "indeterminate form", a terminology related to calculus that you yet need to know of.
Among the many methods of searching what is 0^0, a majority of them led to the conclusion $0^0 = 1$, but because others do not, it is probably safest to say $0^0 = undefined$, which makes $x = 0$ unsuitable for the statement we proved in Part I.
Meanwhile, x cannot equal 0 in Part I of explanation since it would cause an undefined fraction.

Let us switch to the next discussion: Now, what if exponents are fractions?

2: Radicals

Try solving the following problem: for $x^2 = 2$, what is x?

Mathematicians would use the symbol $\sqrt[2]{2}$ to present an exact answer for x.
The definition of this symbol $\sqrt[2]{}$ is such that:

For the solutions of an equation $x^2 = a$, where $x, a \in \mathbb{R}$: $x = \pm\sqrt[2]{a}$ and $\left(\sqrt[2]{a}\right)^2 = x$.
The purpose of the $\pm$ symbol in front of $\sqrt[2]{a}$ is to address that there exist two solutions for a, such that $a = \sqrt[2]{a}\ or - \sqrt[2]{a}$. It is because $\left(-\sqrt[2]{a}\right)^2 = (-1)^2 \times \sqrt[2]{a}^2 = \sqrt[2]{a}^2 = x$.
In fact, the symbol $\sqrt[2]{}$ is more often denoted as $\sqrt{}$.

For example, the solution to equation $x^2 = 4$ would be $x = \pm\sqrt{4}$, which means $x = \pm 2$.
We now know $\sqrt{4} = 2$ and $-\sqrt{4} = -2$ (Be careful $\sqrt{4} \neq -2$). $\sqrt{4} = 2$ because $2^2 = 4$.

So, the solutions x for the equation $x^2 = 2$ is that $x = \pm\sqrt{2}$, which means $x = \sqrt{2}\ or - \sqrt{2}$.
We used or here because x cannot equal to values at the same time, as that would cause the contradiction where a positive value equals its opposite (negative) value.
The symbol $\sqrt[2]{}$ is called a square root, and the symbol $\sqrt{}$ is called a radical.
Any expression including the radical symbol, such as: $\sqrt[2]{}$, $\sqrt[3]{}$, and $\sqrt[4]{}$, is called a radical expression.
Expression $\sqrt[2]{x}$ (which can also be recorded as $\sqrt{x}$) is pronounced "square root of x".
Expression $\sqrt[3]{x}$ would be pronounced "cubic root of x".

We also mentioned that the radical of a number is an "exact form" of the solution for the equation $\boxed{x^2 = 2}$. This is because since the square roots of most numbers are irrational numbers, essentially endless non-repetitive decimal numbers, we cannot write down an exact value of those radicals.
For example, $\sqrt{2}$ is approximated to be 1.414, but its actual value contains infinite amount of digits after decimal point. To address an exact value of the square root of numbers instead of an approximate value, such as 1.414 for the value of $\sqrt{2}$, mathematicians use the symbol $\sqrt{}$ to notate those irrational numbers without having to use decimal points.

In summary, the definition of exact form is the form of expression that accurately represents the specific value of an irrational number.

For equations like $x^2 = a$, $x^5 = a$, or even something like $x^{35} = a$, we can express the solution of those equations in radical expressions of different appearances:

If x^n is defined, then $x^n = a$, $x = \sqrt[n]{a}$.

At the same time, there are many common practices that we need to consider for the radical expressions in the process of simplification: a process of simplifying expressions. This will be introduced in the next section.

For now, we will go back to the question about fractional exponents we left at the end of the last section: what if the exponent is a fraction?
For definition, the square root of a, $\sqrt{a}$, can be expressed as $a^{\frac{1}{2}}$.
Provided that, all fractional exponents follow a pattern:

$\forall\ a, n \in \{(a,n) | a \in \mathbb{R}, a \geq 0 | n \in \mathbb{R}, n \neq 0\}$, $\sqrt[n]{a} = a^{\frac{1}{n}}$.

As there are properties of exponential expressions, there are also properties of radical expressions.
Below is a partial summary of properties:

1. $\forall\ a, n \in \{(a,n) | a \in \mathbb{R}, a \geq 0 | n \in \mathbb{R}, n \neq 0\}$, $\sqrt[n]{a} = a^{\frac{1}{n}}$.
2. $\forall\ a, b, n \in \{(a,b,n) | a, b \in \mathbb{R}, a, b \geq 0 | n \in \mathbb{R}, n \neq 0\}$, $\sqrt[n]{a} + \sqrt[n]{b} \neq \sqrt[n]{a+b}$
3. $\forall\ a, b, n \in \{(a,b,n) | a, b \in \mathbb{R}, a, b \geq 0 | n \in \mathbb{R}, n \neq 0\}$, $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$
4. $\forall\ a, n, k \in \{(a,n,k) | a \in \mathbb{R}, a \geq 0 | n \in \mathbb{R}, n \neq 0 | k \in \mathbb{R} | a^k$ is defined$\}$, $\sqrt[n]{a^k} = \sqrt[n]{a}^{\,k} = a^{\frac{k}{n}}$.

3: Simplification of Expressions

This term is discussed in the last section; simplification refers to the process of simplifying expressions. I'd still like to give an approximate definition to what simplification is, along with an explanation to its importance.

Now, say we have an algebraic expression: $\boxed{\dfrac{\sqrt[3]{27a^4b^{-3}c^2}}{ab^{-1}}}$.
This expression above is complicated, hard to pronounce, and hard to record. To make the expression easier to present, we need to **simplify** the expression by reducing it into a shorter but equivalent expression.
Simplest radical form is an example of simplification. It is the form of radical expression that cannot be reduced shorter. For example, the simplest radical form of $\sqrt{50}$ is $5\sqrt{2}$ and simplest radical form of $\frac{\sqrt{27}}{3}$ is $\frac{3\sqrt{3}}{3}$, which can be simplified into $\sqrt{3}$. Both examples of simplest radical form above is a radical expression unable to be reduced into a shorter radical expression. Meanwhile, as $\sqrt{50} = 5\sqrt{2}$ and $\frac{\sqrt{27}}{3} = \sqrt{3}$, finding the simplest radical form of an expression is also an example of simplification: finding a shorter equivalent version of an expression.

We can simplify expressions until it cannot be reduced into a shorter expression, and the way we determine whether an expression is its shortest version follows:

1. Exponents of all variables are positive integers.
2. One type of variable only exists as a base for once in the expression.
3. Values that sums up to 0 in the final expressions are presented as 0 than its original value.
    a. Ex: The expression $a + 1 - 1$ becomes a, since $1 - 1 = 0$ and does not need to appear in the expression.
4. If the expression is a fraction, the denominator cannot include any radical expressions.

A friendly reminder I'd like to mention before we use an example is that when simplifying an equation, please work from the left-hand-side to the right-hand-side, operate what is in the parentheses first, then work on exponents of terms, and then work at multiplications and divisions, and finally on additions and subtractions.
This order together is named as an oral tip called "PEMDAS" that your instructors may mention in lectures. "PEMDAS" is short for the order: $Parenthesis > Exponents > Multiplication > Division > Addition > Subtraction$, which is the order we simplify an expression and perform arithmetic in.
To clarify, you may also have paid attention to the fact that division is simply multiplication with fractions, and subtraction is addition with negative numbers, so that there is no actual priority between multiplication and division, and there is no priority between addition and subtraction as well.

Anyways, below is our example of simplification. We will be simplifying a much complicated expression so we can demonstrate maximum steps in simplifying an expression.

We will now simplify the expression: $\boxed{\frac{\sqrt[3]{27a^4b^{-3}c^2}}{ab^{-1}\sqrt[3]{2}}}$.
Step 1: Because the expression is a fraction, it may be wiser to deal with one part of the fraction first. We will first deal with the numerator by finding the simplest radical form of that expression $\sqrt[3]{27a^4b^{-3}c^2}$. $\sqrt[3]{27a^4b^{-3}c^2} = \sqrt[3]{27} \times \sqrt[3]{a^4} \times \sqrt[3]{b^{-3}} \times \sqrt[3]{c^2} = 3 \times a\sqrt[3]{a} \times b^{-1} \times \sqrt[3]{c^2}$.
If you are confused by the above arithmetic, you can think of the above expression with fractional exponents.
$\sqrt[3]{27a^4b^{-3}c^2} = (27a^4b^{-3}c^2)^{\frac{1}{3}} = (27)^{\frac{1}{3}} \times (a^4)^{\frac{1}{3}} \times (b^{-3})^{\frac{1}{3}} \times (c^2)^{\frac{1}{3}} = 3 \times a \times (a)^{\frac{1}{3}} \times b^{-1} \times c^{\frac{2}{3}}$.

Step 2: Eliminate, or "cross out", terms with similar base from denominator and numerator.
The current expression has turned into $\frac{3ab^{-1}\times\sqrt[3]{ac^2}}{ab^{-1}\sqrt{2}}$ or $\frac{3ab^{-1}\times a^{\frac{1}{3}}c^{\frac{2}{3}}}{ab^{-1}2^{\frac{1}{2}}}$ if you use fractional exponents.
Either way it ends up being simplified into $\frac{3\sqrt[3]{ac^2}}{\sqrt[3]{2}}$, equivalent to $3a^{\frac{1}{3}}c^{\frac{2}{3}}2^{-\frac{1}{3}}$.
The reason we did not convert b^{-1} in both denominator and numerator of fraction is because they are going to be eliminated either way.

Then we will deal with the harder part of simplification:

Step 3: Now we determine whether the radical expression is in its simplest form. If you are using fractional exponents, you have to convert it to radical form so that your expression only has positive integer exponents. The $\sqrt[3]{2}$ at the bottom prevented our expression $\frac{3\sqrt[3]{ac^2}}{\sqrt[3]{2}}$ from being the simplest radical form, so we must find a method to eliminate the $\sqrt[3]{2}$.
This can be done by multiplying both the denominator and numerator by $\sqrt[3]{2}^2$, which would make the denominator $\sqrt[3]{2}^3 = 2$, and make the numerator $3\sqrt[3]{ac^2} \times \sqrt[3]{4} = 3\sqrt[3]{4ac^2}$.
Step 4: Because the radical expression $\frac{3\sqrt[3]{4ac^2}}{2}$ cannot be simplified further (based on above guidelines), $\frac{\sqrt[3]{27a^4b^{-3}c^2}}{ab^{-1}\sqrt[3]{2}}$'s shortest simplified form is $\boxed{\frac{3\sqrt[3]{4ac^2}}{2}}$.

Again, to familiarize with the processes of simplification needs practice.
For cases where an expression is a fraction with a denominator of radical expressions, we need to convert the denominator into a non-radical expression. The action of converting a fraction with a denominator of radical expression into a fraction with a denominator of non-radical expression is known as "**rationalization**".

Case 1: Simplify $\frac{4}{\sqrt{5}}$.
This one is relatively easier. To make the denominator a non-radical expression, multiply the entire fraction by $\frac{\sqrt{5}}{\sqrt{5}}$, which is equal to 1.
The entire multiplication looks like this: $\frac{4}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5}}{5}$.
Here, although the original expression is shorter than its simplified version, as the denominator of a fraction cannot be a radical expression, it has to be simplified into the expression above, which happens to be the most simplified version of that equivalent expression.

Case 2: Simplify $\frac{4}{\sqrt{5}-1}$.
This one is relatively harder. To make the denominator a non-radical expression, multiply the entire fraction by $\frac{(\sqrt{5}+1)}{(\sqrt{5}+1)}$, which is equal to 1.
The entire multiplication looks like this: $\frac{4}{\sqrt{5}-1} \times \frac{(\sqrt{5}+1)}{(\sqrt{5}+1)} = \frac{4(\sqrt{5}+1)}{5-1} = (\sqrt{5}+1)$.

The above example requires knowledge on how to multiply expressions together, so you are not expected to understand Case 2 of rationalization now.
We will introduce relevant knowledges, which is "Algebraic Shortcuts", in the next chapter.
By the way, please consider familiarizing yourself with fractional exponents. They will come up again in significant amounts within two years you finish Algebra II.

Practice Questions Before 10-2 (No CALC)

Part I. Provide the solution to the given equations in (a) simplified radical form (with no radical denominators) and (b) exponential form, and (optional) take an approximate value of your solution if the denominator of your solution's exponent in exponential form is a power of 2 and compare it with your calculator.
*We will revisit your Answer (c)s from this part in Volume IV when we describe Calculus.
*You don't need to include the $\pm$ sign for this part.

Equation	Answer (a)	Answer (b)	Answer (c;optional)
(1) $x^2 = 2$	$\sqrt{2}$	$2^{\frac{1}{2}}$	1.414
(2) $x^2 = 3$	$\sqrt{3}$	$3^{\frac{1}{2}}$	1.732
(3) $x^2 = 4$	2	2	2
(4) $x^2 = 0.6$	$\frac{\sqrt{15}}{5}$	$0.6^{\frac{1}{2}}$	0.775
(5) $x^2 = 0.9$	$\frac{3\sqrt{10}}{10}$	$0.9^{\frac{-1}{2}}$	0.949
(6) $x^2 = 10$	$\sqrt{10}$	$10^{\frac{1}{2}}$	3.162
(7) $x^3 = 5$	$\sqrt[3]{5}$	$5^{\frac{1}{3}}$	1.701
(8) $x^4 = 5$	$\sqrt[4]{5}$	$5^{\frac{1}{4}}$	1.495
(9) $x^8 = 125$	$\sqrt[8]{125}$	$5^{\frac{3}{8}}$	1.829
(10) $x^{16} = 25$	$\sqrt[8]{5}$	$5^{\frac{1}{8}}$	1.223
(11) $x^{0.5} = 49$	2401	2401	2401
(12) $x^{-0.5} = 49$	$\frac{1}{2401}$	2401^{-1}	0.000416
(13) $x^{-4} = 49$	$\frac{\sqrt{7}}{7}$	$7^{-\frac{1}{2}}$	0.378
(14) $x^4 = 49$	$\sqrt{7}$	$7^{\frac{1}{2}}$	2.646
(15) $3x^{2.5} = \sqrt{72}$	$\sqrt[5]{8}$	$2^{\frac{3}{5}}$	1.515
(16) $5x^{-1.25} = 10\sqrt{8}$	$\frac{1}{4}$	2^{-2}	0.24
(17) $10x^{10} = \sqrt{1024000}$	$2\sqrt[10]{10}$	$2 \times 10^{\frac{1}{10}}$	2.518
(18) $x^2 = 6561$	81	81	81
(19) $x^7 = 8748$	$3\sqrt[7]{4}$	$3 \times 2^{\frac{2}{7}}$	3.657
(20) $x^4 = 298116$	$\sqrt{546}$	$546^{\frac{1}{2}}$	23.367

Part II. Rewrite the following expressions as (a) exponential form, (b) simplified radical form (with no radical denominators), and (c) simplify the given expressions.

Expression	Answer (a)	Answer (b)	Answer (c)

(1) $\sqrt[3]{\mathbf{90x^3y^5}}$	$90^{\frac{1}{3}}xy^{\frac{5}{3}}$	$xy\sqrt[3]{\mathbf{90y^2}}$	$xy\sqrt[3]{\mathbf{90y^2}}$
(2) $\sqrt[4]{\mathbf{180x^7y^{-5}}}$	$\frac{180^{\frac{1}{4}}x^{\frac{7}{4}}}{y^{\frac{5}{4}}}$	$\frac{x\sqrt[4]{180x^3y^3}}{y^2}$	$\frac{x\sqrt[4]{180x^3y^3}}{y^2}$
(3) $\sqrt[2]{\mathbf{1859x^{-13}y^{15}}}$	$\frac{13\times 11y^{\frac{15}{2}}}{x^{\frac{13}{2}}}$	$\frac{13\sqrt{11xy}}{x^7y^7}$	$\frac{13\sqrt{11xy}}{x^7y^7}$
(4) $\sqrt[2]{\mathbf{52x^7y^3z^2}}$	$2\times 13^{\frac{1}{2}}x^{\frac{7}{2}}y^{\frac{3}{2}}z$	$2x^3yz\sqrt{13xy}$	$2x^3yz\sqrt{13xy}$
(5) $\sqrt[3]{\mathbf{27x^{-5}y^{-3}z^{147}}}$	$\frac{3z^{49}}{x^{\frac{5}{3}}y}$	$\frac{3z^{49}\sqrt[3]{x}}{x^2y}$	$\frac{3z^{49}\sqrt[3]{x}}{x^2y}$
(6) $\frac{\sqrt[3]{\mathbf{90x^3y^5}}}{\mathbf{xy^2}}$	$\frac{90^{\frac{1}{3}}}{y^{\frac{1}{3}}}$	$\frac{\sqrt[3]{\mathbf{90y^2}}}{y}$	$\frac{\sqrt[3]{\mathbf{90y^2}}}{y}$
(7) $\frac{\sqrt[3]{\mathbf{540x^3y^{-5}z^4}}}{\mathbf{24xy^{-3}z}}$	$\frac{20^{\frac{1}{3}}y^{\frac{4}{3}}z^{\frac{1}{3}}}{8}$	$\frac{y\sqrt[3]{20yz}}{8}$	$\frac{y\sqrt[3]{20yz}}{8}$
(8) $\frac{\sqrt[4]{\mathbf{567x^4y^5}}}{\mathbf{45xz}}$	$\frac{7^{\frac{1}{4}}x^{\frac{1}{4}}y^{\frac{3}{2}}}{15z}$	$\frac{y\sqrt[4]{7xy^2}}{15z}$	$\frac{y\sqrt[4]{7xy^2}}{15z}$
(9) $\mathbf{25^{1.5}}\times\sqrt{\mathbf{75}}$	$625\times 3^{\frac{1}{2}}$	$625\sqrt{3}$	$625\sqrt{3}$
(10) $\frac{\mathbf{25^{1.5}}\times\sqrt{\mathbf{175}}}{\sqrt{\mathbf{35}}}$	$125\times 5^{\frac{1}{2}}$	$125\sqrt{5}$	$125\sqrt{5}$
(11) $\mathbf{3^{0.2}}\times\mathbf{4^{0.1}}\times\mathbf{5\sqrt{5}}$	$5\times 112500^{\frac{1}{10}}$	$5\sqrt[10]{112500}$	$5\sqrt[10]{112500}$
(12) $\mathbf{2^{0.2}}\times\mathbf{3^{4.5}}\times\sqrt{\mathbf{1.44}}$	$97.2\times 486^{\frac{1}{5}}$	$97.2\sqrt[5]{486}$	$97.2\sqrt[5]{486}$
(13) $\mathbf{7^{0.5\sqrt[5]{32}}}\times\mathbf{21^{0.2\sqrt[2]{9}}}$	$567^{\frac{1}{500}}$	$\sqrt[500]{567}$	$\sqrt[500]{567}$
(14) $\mathbf{169^{7.5}}\times\mathbf{13^{\frac{1}{\lvert\sqrt{225}\rvert}}}$	$13^{\frac{226}{15}}$	$13\sqrt[15]{13}$	$13\sqrt[15]{13}$
(15) $\frac{\mathbf{225^{0.25}}}{\mathbf{15\sqrt{2}}}$	$\frac{1}{30^{\frac{1}{2}}}$	$\frac{\sqrt{30}}{30}$	$\frac{\sqrt{30}}{30}$

Practice Questions After 10-2 (No CALC)

Part I. Rationalize and simplify below expressions, in terms of the variables given.

Expression	Rationalization	Expression	Rationalization
(1) $\frac{\mathbf{1}}{\sqrt{\mathbf{x}}\mathbf{-1}}$	$\frac{\sqrt{x}+1}{x-1}$	(2) $\frac{\sqrt{x}-1}{\sqrt{x}+1}$	$\frac{(\sqrt{x}-1)(\sqrt{x}-1)}{x-1}$
(3) $\frac{\sqrt{\mathbf{x+1}}}{\sqrt{\mathbf{x^2-1}}}$	$\frac{\sqrt{x}-1}{x-1}$	(4) $\frac{1}{\sqrt{x}+1}$	$\frac{\sqrt{x}-1}{x-1}$
(5) $\frac{\mathbf{1}}{\mathbf{2x+\sqrt[3]{4}}}$	$\frac{2x^2-x\sqrt[3]{4}+x\sqrt[3]{2}}{2(2x^3+1)}$	(6) $\frac{1}{2x-\sqrt[3]{4}}$	$\frac{2x^2+x\sqrt[3]{4}+x\sqrt[3]{2}}{2(2x^3-1)}$

(7) $\frac{\sqrt[5]{6a^7b^{-3}c}}{\sqrt[5]{54ab^9c^4}}$	$\frac{a\sqrt[5]{27ab^3c^2}}{3b^3c}$	(8) $\frac{\sqrt[4]{2a^3b^{11}}}{\sqrt[4]{128a^{12}b^7}}$	$\frac{b\sqrt[4]{4a^3}}{4a^3}$

Part II. Simplify the following expressions and write down your process of simplification:

Expression	Solution
(1) $\frac{2}{2+x}+\frac{5}{x-2}+\frac{1}{x^2-4}$	$\frac{7x+7}{x^2-4}$
(2) $\frac{5}{x-5}+\frac{25}{x^2+5x+25}$	$\frac{5x^2+50x}{x^3-125}$
(3) $\frac{19}{x^2-11x+28}-\frac{4}{x^2-2x-8}$	$\frac{15x+66}{x^3-9x^2+6x+56}$
(4) $\frac{x^2+x-20}{x^2+5x+25}\div\frac{x^3+125}{x^4+25x^2+625}$	$x-4$
(5) $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x-y}}\div\frac{x^3-y^3}{\sqrt{x}+\sqrt{y}}$	$\frac{\sqrt{x-y}}{x^3-y^3}$
(6) $\frac{x^2(x+3y)+y^2(3x+y)}{\sqrt[3]{(2x+2y)^8}}$	$\frac{\sqrt[3]{2x+2y}}{8}$
(7) $\left(\frac{ab^{-3}}{b^2c^3}\right)^{x+2}\left(\frac{ab^3c^2}{b^5}\right)^{3-x}$	$\frac{a^5}{b^{3x+16}c^{5x}}$
(8) $\frac{\frac{x+2}{x}-\frac{a+2}{a}}{x-a}$	$-\frac{2}{xa}$
(9) $\frac{6-\frac{5}{k}}{1+\frac{5}{k^2}}$	$\frac{k(6k+5)}{k^2+5}$
(10) $\frac{x^2y^{-2}-y^2x^{-2}}{xy^{-1}-yx^{-1}}$	$\frac{x^2+y^2}{xy}$
(11) $\sqrt[3]{250y^2}+\sqrt[3]{\frac{y^2}{2}}$	$5\sqrt[3]{2y^2}+\frac{\sqrt[3]{4y^2}}{2}$
(12) $\left(\frac{n^{2x+y}}{n^{x-y}}\right)^3$	n^{3x+6y}